Adjustments to the law of the wall above an Amazonian Forest explained by a spectral link

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Introduction

- The significance of the *roughness sublayer* (RSL) to a plethora of physical, chemical, and biological processes is not in dispute.

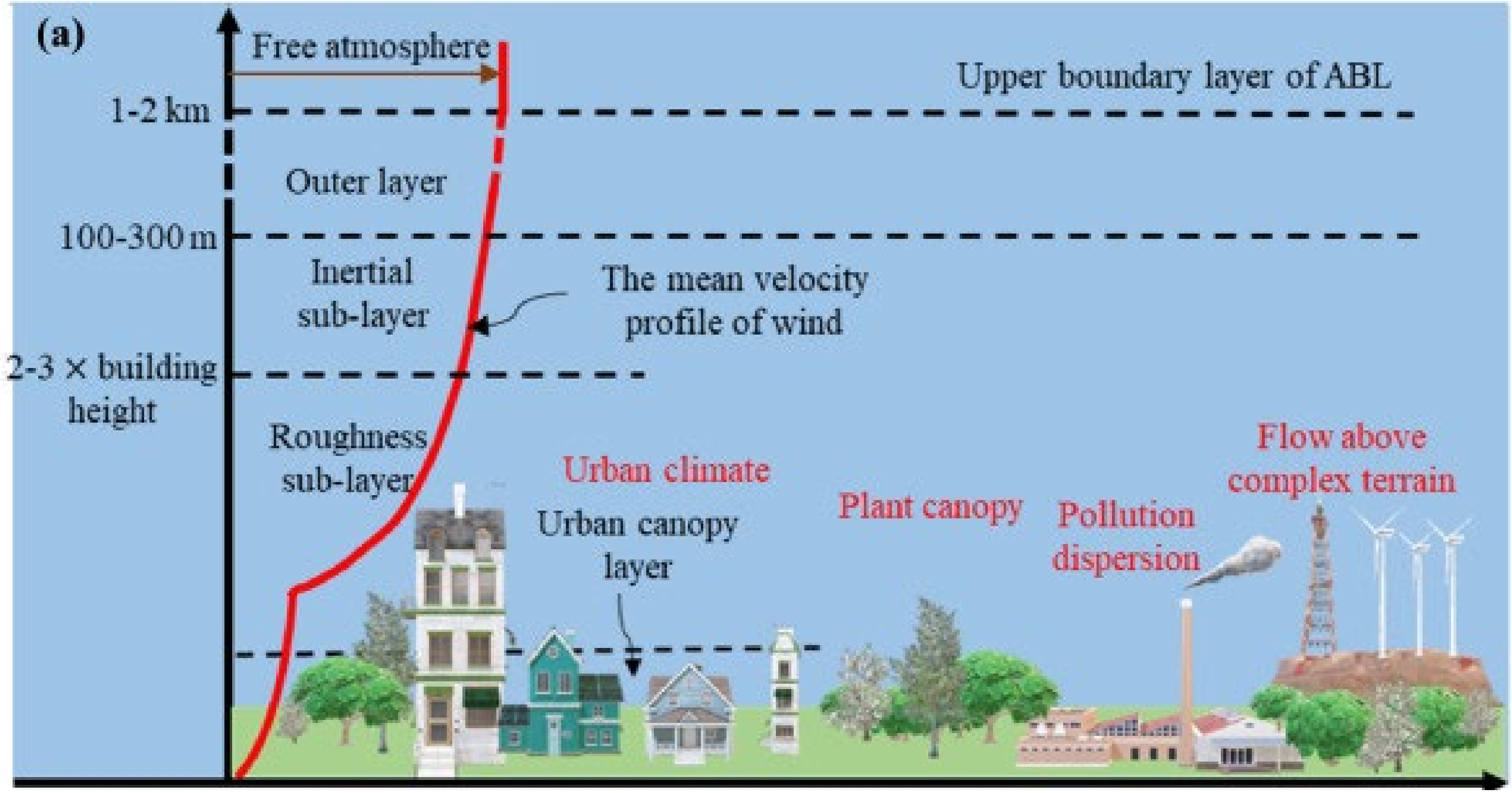


Figure from: https://doi.org/10.1016/j.ijft.2021.100077

Introduction

Focus here on **RSL ABOVE CANOPIES** – where *multiple eddy types* dominate biosphere-atmosphere exchange

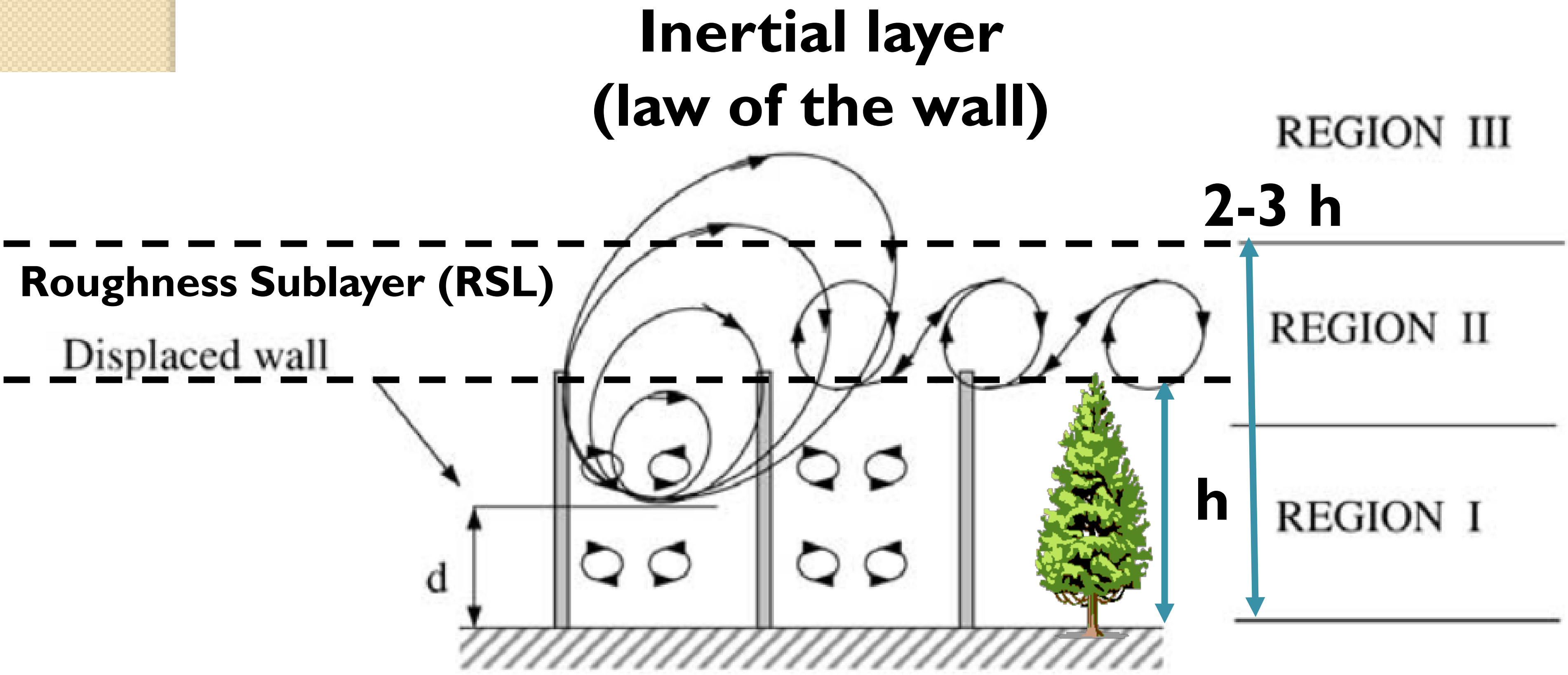
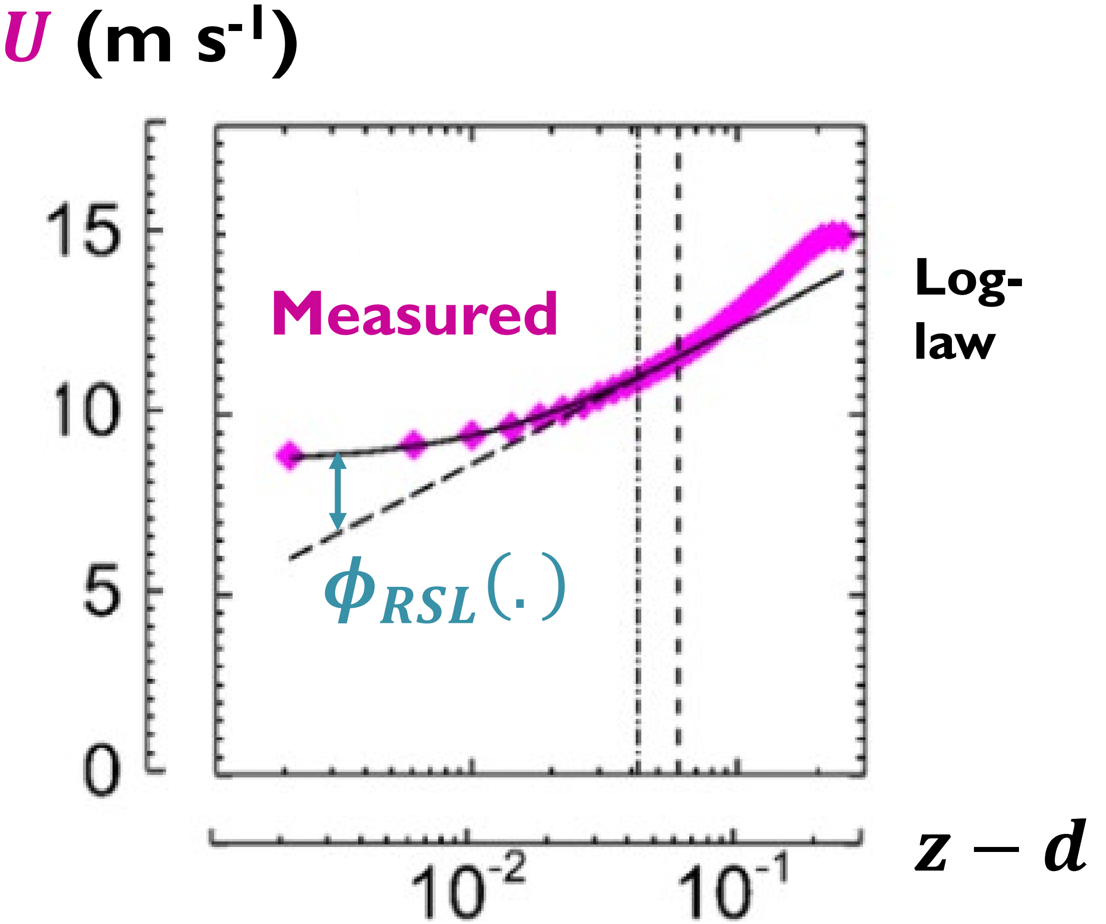


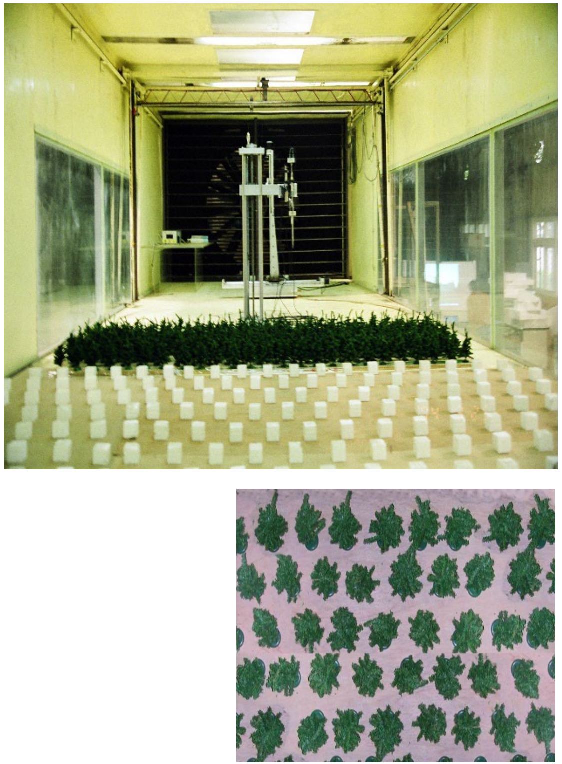
Figure revised from Poggi et al. (2004)

Roughness sublayer correction

- Correction **increases** mean velocity U relative to its log-law extrapolation



Wind tunnel experiments



Courtesy Chia-Ren Chu

Roughness sublayer

- Correction to the ‘law of the wall’:

$$\frac{dU}{dz}\frac{\kappa(z-d)}{u_*} = \phi_{RSL}\left(\frac{z}{h}, \ldots\ldots\right)$$

- Log-law recovered when $\phi_{RSL}(.) = 1$

- Current approach:

Empirical coefficient

$$\phi_{RSL}(.) = 1 - exp\left[-a\left(\frac{z-d}{z^*}\right)\right]$$

Thickness of the RSL

Problem:

- Many ϕ_{RSL} models proposed - but miss the key mechanism they purported to represent- **energetics of eddies**.

Objective:

- Derive ϕ_{RSL} from energetics of turbulent eddies and compare with experiments in non-ideal settings (e.g. Amazonia).

Model Co-spectra from simplified budget

<u>Eddy sizes or scales</u> considered using the co-spectrum:

TURBULENT STRESS

$$\overline{w'u'} = \int_0^\infty F_{wu}(k)dk$$

1-D Wavenumber
(inverse eddy size)

Molecular viscosity

Production by mean gradients

Pressure Re-distribution

$$\cancelto{0}{\frac{\partial F_{wu}(k)}{\partial t}} + 2\nu k^2 F_{wu}(k) = 0 = \frac{dU}{dz} F_{ww}(k) + T_{wu}(k) + \pi_u(k)$$

Viscous
dissipation
destroying
$\overline{u'w'}$

Vertical velocity spectrum

Transfer by
turbulence
across scales

Two-term co-spectral budget

Simplify: High Reynolds number, stationary and equilibrium co-spectral budgets (i.e. turbulent transfer terms ignored).

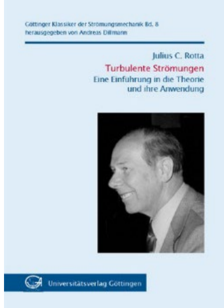
Production of covariance $F_{wu}(k)$ at scale k:

$$\frac{dU}{dz} F_{ww}(k) + \pi_u(k) = 0$$

De-correlation due to pressure - velocity interaction (**requires closure at k**)

A Rotta type closure for $\pi_u(k)$

Models the *universal* tendency of **all** turbulent flows to return to isotropy at small scales


Göttinger Klassiker der Strömungsmechanik Bd. 8
herausgegeben von Andreas Dillmann

Julius C. Rotta
Turbulente Strömungen
Eine Einführung in die Theorie
und ihre Anwendung

Universitätsverlag Göttingen

Classical
Rotta term
SLOW PART

Isotropization
of the production
FAST PART

$$\pi_u(k) = -A_u \frac{F_{wu}(k)}{\tau(k)} - C_{Iu} \boxed{\frac{dU}{dz} F_{ww}(k)}$$

Production by
mean gradients

$\tau(k)$ = Onsager (1948) relaxation time scale

A_u = Rotta constant (~1.8).

C_{Iu} = Coefficient related to isotropization of production

Rapid Distortion Theory predicts C_{Iu} = 3/5 (Pope, 2000).

Stationary and equilibrium solution to the two-term co-spectral budget

This simplified budget relates the **co-spectrum** to the vertical velocity spectrum $F_{ww}(k)$.

$$F_{wu}(k) = \frac{1 - C_{IU}}{A_u}\left[\frac{dU}{dz}F_{ww}(k)\right]\tau(k)$$

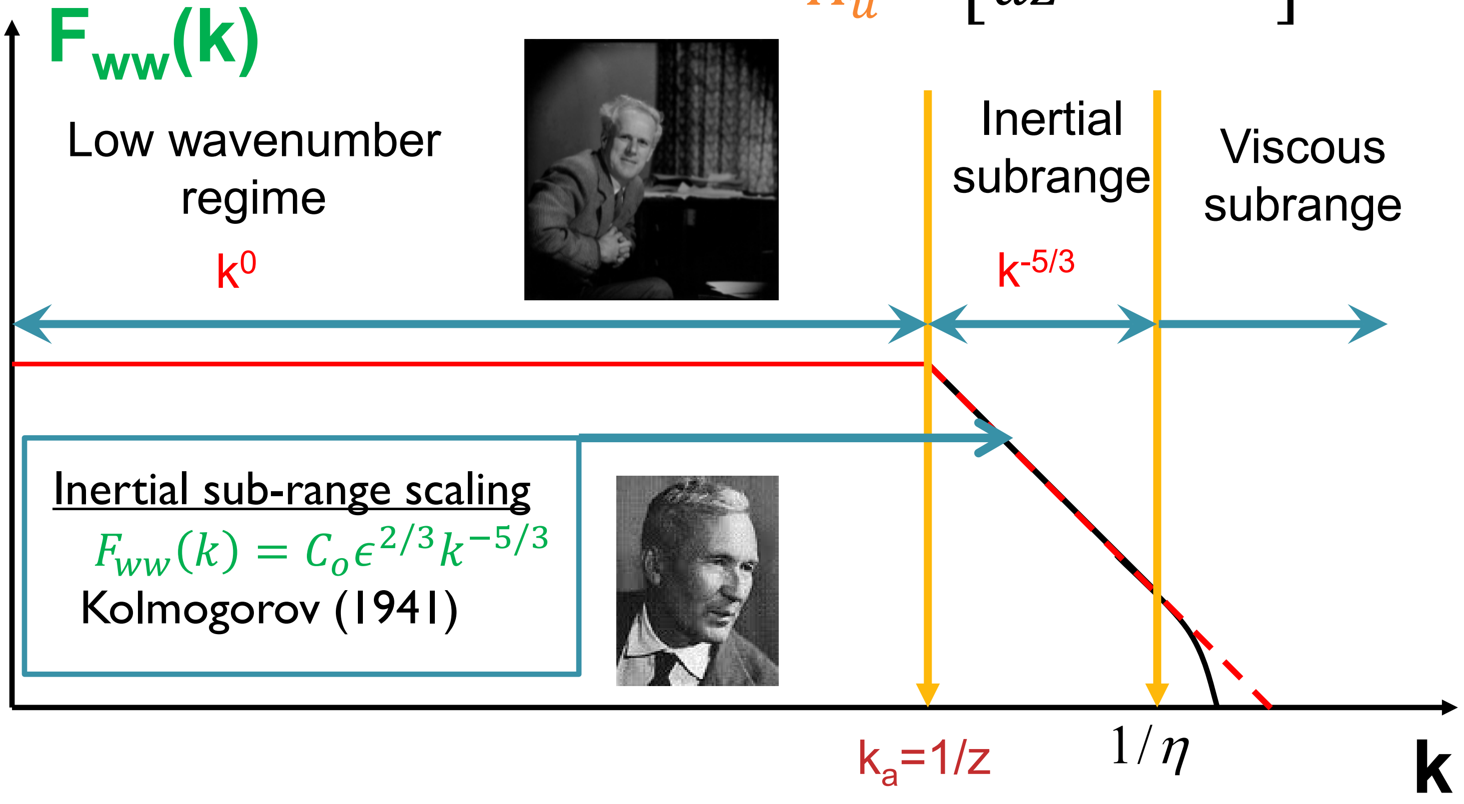
$\tau(k)$ = Onsager's (1948) relaxation time scale = $k^{-2/3}\epsilon^{-1/3}$

$F_{ww}(k)$ = Vertical velocity energy spectrum,
must be externally supplied (measured or assumed)

Shape of $F_{ww}(k)$

For canonical boundary layers (in the inertial layer):

$$F_{wu}(k) = \frac{1 - C_{IU}}{A_u}\left[\frac{dU}{dz} F_{ww}(k)\right]\tau(k)$$



$F_{ww}(k)$

Low wavenumber regime

k^0

Inertial subrange

$k^{-5/3}$

Viscous subrange

Inertial sub-range scaling

$$F_{ww}(k) = C_o \epsilon^{2/3} k^{-5/3}$$

Kolmogorov (1941)

$k_a=1/z$

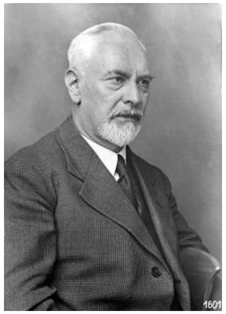
$1/\eta$

k

Albert Townsend's hypothesis

The integrated co-spectral budget

$$\frac{dU}{dz} = \left(\frac{4}{7}\frac{1}{C_o}\frac{A_u}{1-C_{IU}}\right)^{3/4}\frac{\overline{-(w'u')}^{1/2}}{z},$$



Upon integration yields the **Prandtl-von Karman** velocity profile:

$$U(z) = \left(\frac{4}{7}\frac{1}{C_o}\frac{A_u}{1-C_{IU}}\right)^{\frac{3}{4}} u_* \log(z) + B$$

The von Karman constant can be estimated

$$\kappa = \left(\frac{4}{7}\frac{1}{\underset{(24/55)(1.5)}{C_o}}\frac{\overset{1.8}{\overset{\downarrow}{A_u}}}{1-\underset{3/5}{C_{IU}}}\right)^{-\frac{3}{4}} \approx 0.36$$

Estimating RSL correction

DETAILED MODEL - Measured $F_{ww}(k)$

$$\phi_{RSL}(.) = \left(-\frac{\overline{u'w'}}{{u_*}^2}\right)\left(\frac{A_u}{1-C_{IU}}\right)\frac{u_*\kappa(z-d)}{\int_0^\infty \tau(k)\ F_{ww}(k)dk}$$

SIMPLIFIED MODEL:

$$\phi_{RSL}(.) = \left(-\frac{\overline{u'w'}}{{u_*}^2}\right)\left(\frac{A_u}{1-C_{IU}}\right)\frac{u_*\kappa(z-d)}{\tau_{eff}\,\sigma_w^2}; \quad \tau_{eff} = \frac{2\sigma_w^2}{\epsilon(z)}$$

Emergence of a 'macro-dissipation' length in $\phi_{RSL}(.)$

$$\phi_{RSL}(.) = \frac{1}{2}\left(-\frac{\overline{u'w'}}{{u_*}^2}\right)\left(\frac{A_u}{1 - C_{IU}}\right)\left(\frac{u_*}{\sigma_w}\right)^4 \frac{L_{BL}}{L_d};$$

$$L_{BL} = \kappa(z - d);\ L_d = \frac{{u^*}^3}{\epsilon(z)}.$$

This is a new scale for RSL that differs from the 'canonical' shear length scale derived from *mixing layer analogy* (L_s) put forth by Raupach, Finnigan and others.

$$L_s = \frac{U}{dU/dz}$$

Experiments

23 March 2014 to 16 January 2015

GOAmazon (K34)

25 October to 25 November of 2015

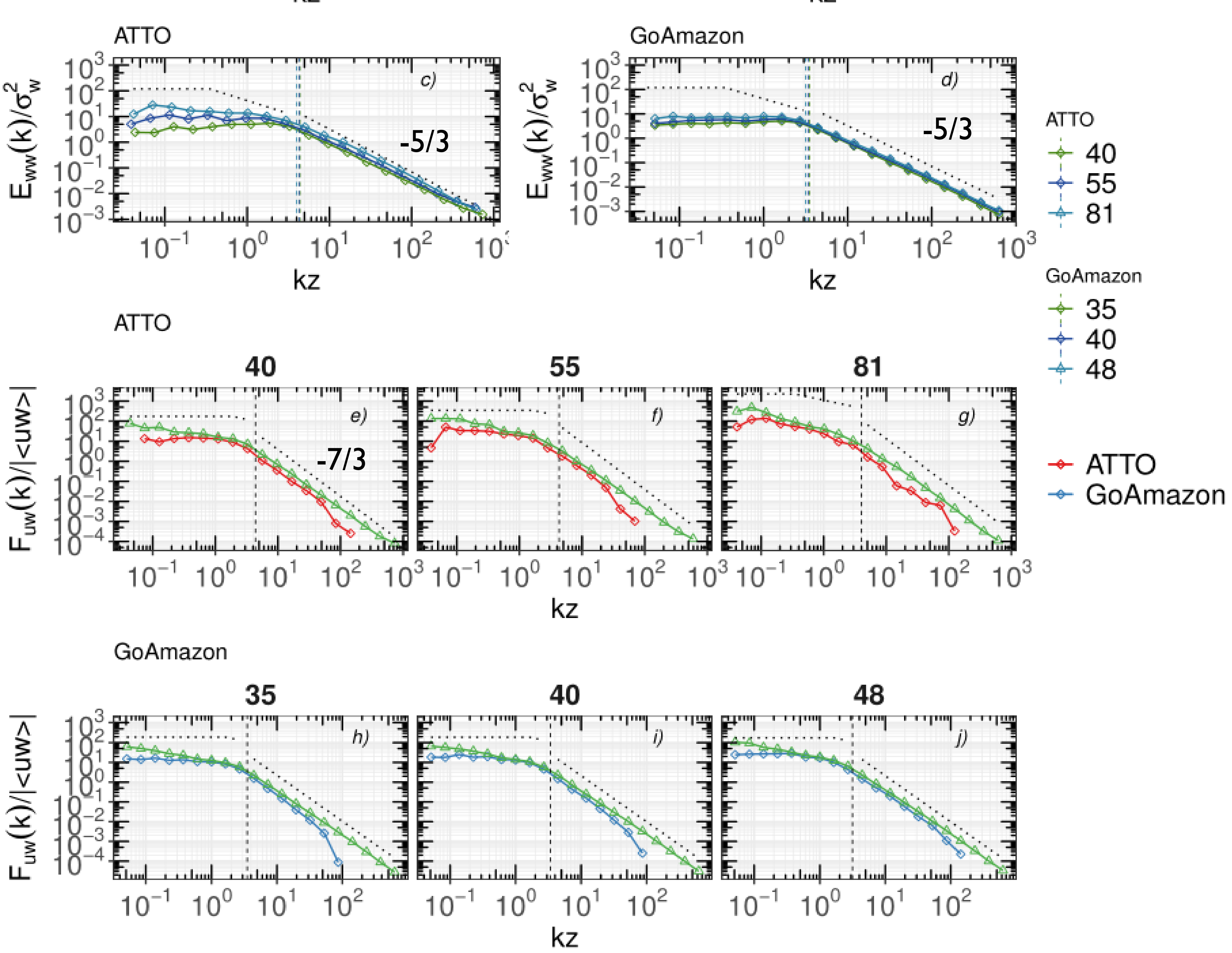
ATTO

Amazon Tall Tower Observatory

At both sites: h=35 m and LAI = 6

GoAmazon: From Fuentes et al. (2016)

ATTO: https://commons.wikimedia.org/wiki/File:Amazon_Tall_Tower_Observatory.jpg


ATTO
GoAmazon
c)
d)
-5/3
-5/3
$E_{ww}(k)/\sigma_w^2$
kz
ATTO
40
55
81
GoAmazon
35
40
48
ATTO
40
55
81
e)
f)
g)
-7/3
$F_{uw}(k)/|<uw>|$
kz
ATTO
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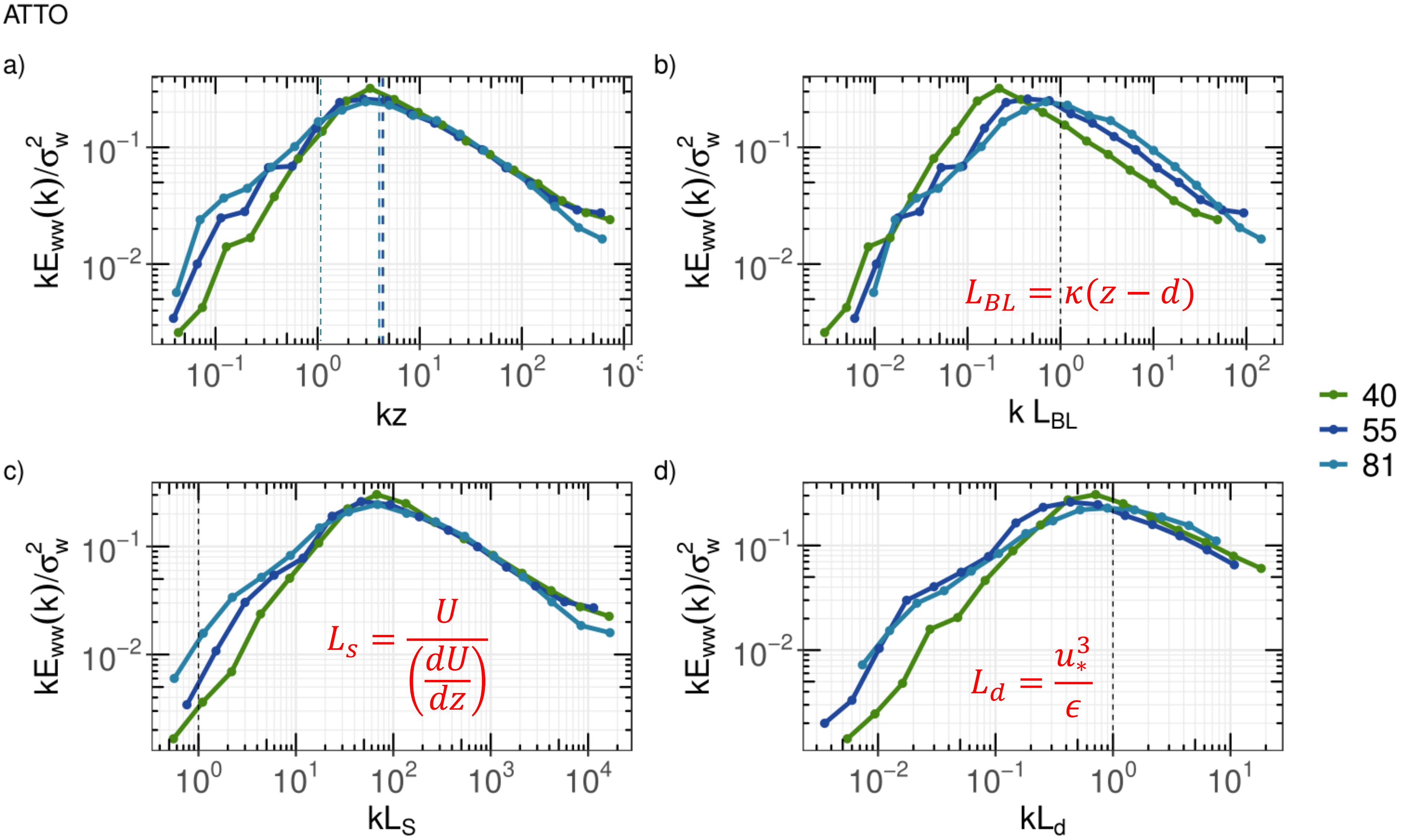
35
40
48
h)
i)
j)
$F_{uw}(k)/|<uw>|$
kz

Breakpoints in spectra (ATTO)



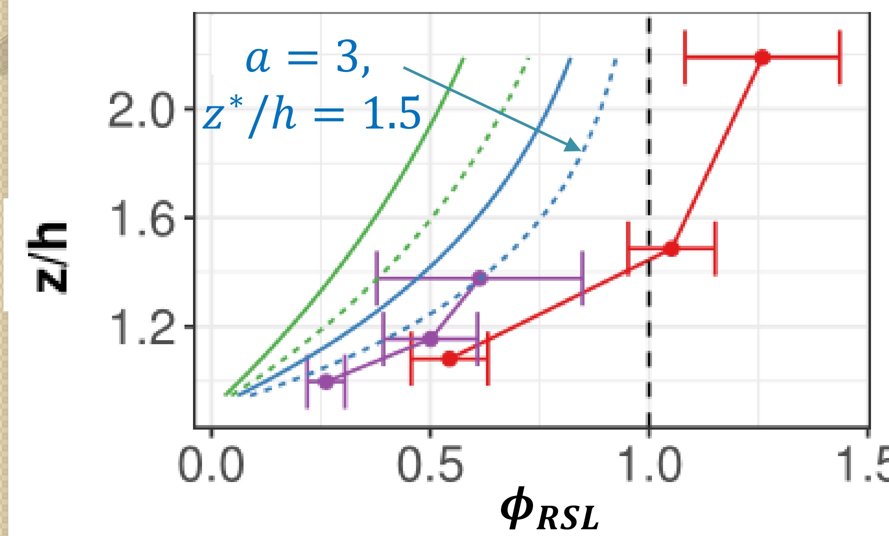
$$L_{BL} = \kappa(z - d)$$

$$L_s = \frac{U}{\left(\frac{dU}{dz}\right)}$$

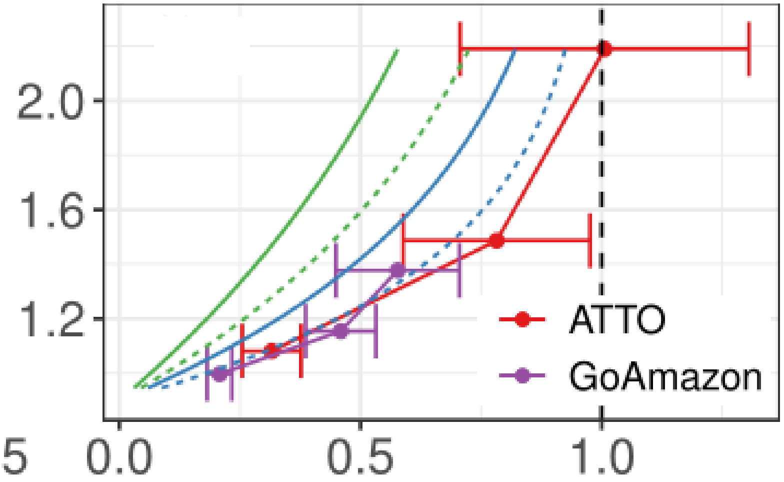
$$L_d = \frac{u_*^3}{\epsilon}$$

Model comparisons

$$\frac{dU}{dz}\frac{\kappa(z-d)}{u_*} = \phi_{RSL}$$



DETAILED MODEL
Uses measured $F_{ww}(k)$

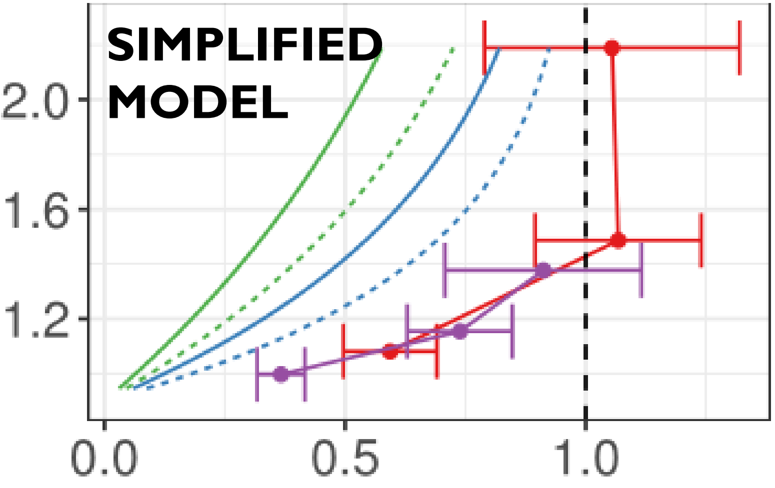


Lines-EMPIRICAL MODEL

with $a \in [2,3]$ **and** $\frac{z^*}{h} \in [1.5, 3]$

spanning the literature for forests

$$\phi_{RSL}(.) = 1 - exp\left[-a\left(\frac{z-d}{z^*}\right)\right]$$


SIMPLIFIED MODEL

Conclusions

- An eddy viscosity that accommodates energetics of turbulence - analogous to the fluctuation-dissipation theorem in statistical mechanics

$$\nu_t = \left(\frac{1 - C_{IU}}{A_u}\right)\int_0^\infty \tau(k)\, F_{ww}(k)dk.$$

- Emergence of a macro-scale dissipation length (L_d) that explains transitions in $F_{ww}(k)$ as well as RSL correction functions ($\boldsymbol{\phi_{RSL}}$).

- Derived $\boldsymbol{\phi_{RSL}}$ appears robust to non-ideal conditions at the two forested sites in Amazonia.

- Future work: include thermal stratification

EXTRA SLIDES

Integration of co-spectrum across all k

$$F_{wu}(k) = \frac{1 - C_{IU}}{A_u}\left[\frac{dU}{dz} F_{ww}(k)\right]\tau(k)$$

Integration limit applicable for $Re_* \rightarrow \infty, \eta \rightarrow 0$

$$\overline{w'u'} = \int_0^\infty F_{wu}(k)dk = \int_0^{k_a} F_{wu}(k)dk \qquad + \qquad \int_{k_a}^\infty F_{wu}(k)dk$$

Large scales (attached)

$F_{ww}(k) = C_o \epsilon^{2/3} {k_a}^{-5/3}$

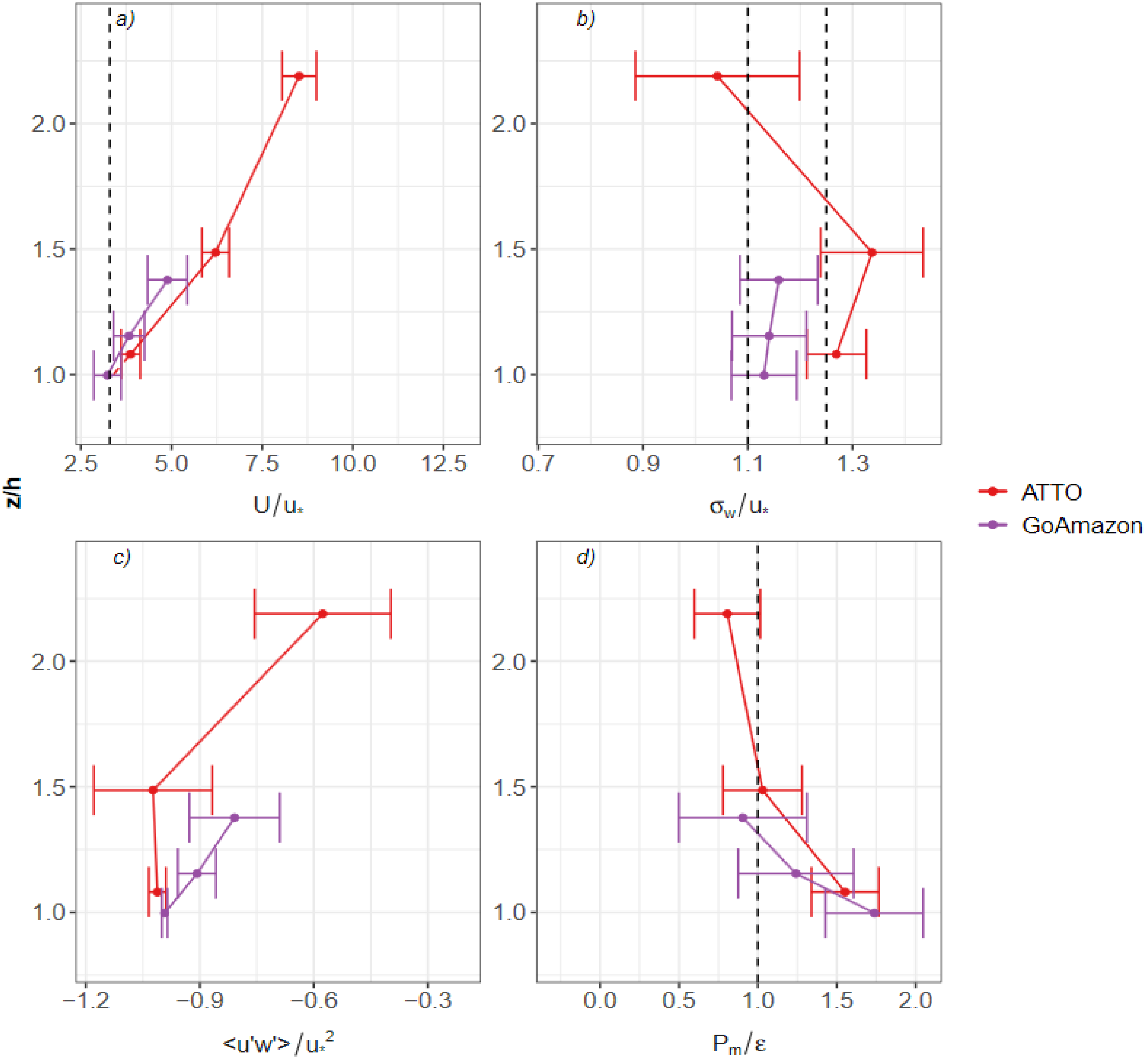
Inertial scales (detached)

$F_{ww}(k) = C_o \epsilon^{2/3} k^{-5/3}$

Assume turbulent kinetic energy (TKE) budget is in equilibrium so that

$$\epsilon \quad = \quad -\overline{w'u'}\frac{dU}{dz}$$

Dissipation of TKE = Production of TKE


a)
b)
c)
d)
z/h
2.0
1.5
1.0
2.5
5.0
7.5
10.0
12.5
U/u*
0.7
0.9
1.1
1.3
σw/u*
−1.2
−0.9
−0.6
−0.3
<u'w'>/u*²
0.0
0.5
1.0
1.5
2.0
Pm/ε
ATTO
GoAmazon

Solution for the inertial subrange

(known in ISR)

$$\boxed{F_{ww}(k) = C_o \epsilon^{2/3} k^{-5/3}}$$

$$\boxed{\tau(k) = k^{-2/3} \epsilon^{-1/3}}$$

$$F_{wu}(k) = \frac{1 - C_{IU}}{A_u}\left[\frac{dU}{dz} F_{ww}(k)\right] \tau(k)$$

$$F_{wu}(k) = \left(C_o \frac{1 - C_{IU}}{A_u}\right)\left[\frac{dU}{dz}\right] \epsilon^{1/3} k^{-7/3}$$

NOTES:

If spectrum of *vertical velocity* scales as **-5/3**, then the co-spectrum scales as **-7/3** – consistent with experiments and Lumley's (1967) arguments.

Also suggestive that turbulent transfer term $T_{wu}(k)$ may be less important if a -7/3 power-law prevails in co-spectrum.

Comparison to Lumley (1967)

$$F_{wu}(k) = C_{uw}\left[\frac{dU}{dz}\right]\epsilon^{1/3}k^{-7/3}$$

Lumley’s result
Dimensional analysis

$$F_{wu}(k) = \left(C_o\frac{1 - C_{IU}}{A_u}\right)\left[\frac{dU}{dz}\right]\epsilon^{1/3}k^{-7/3}$$

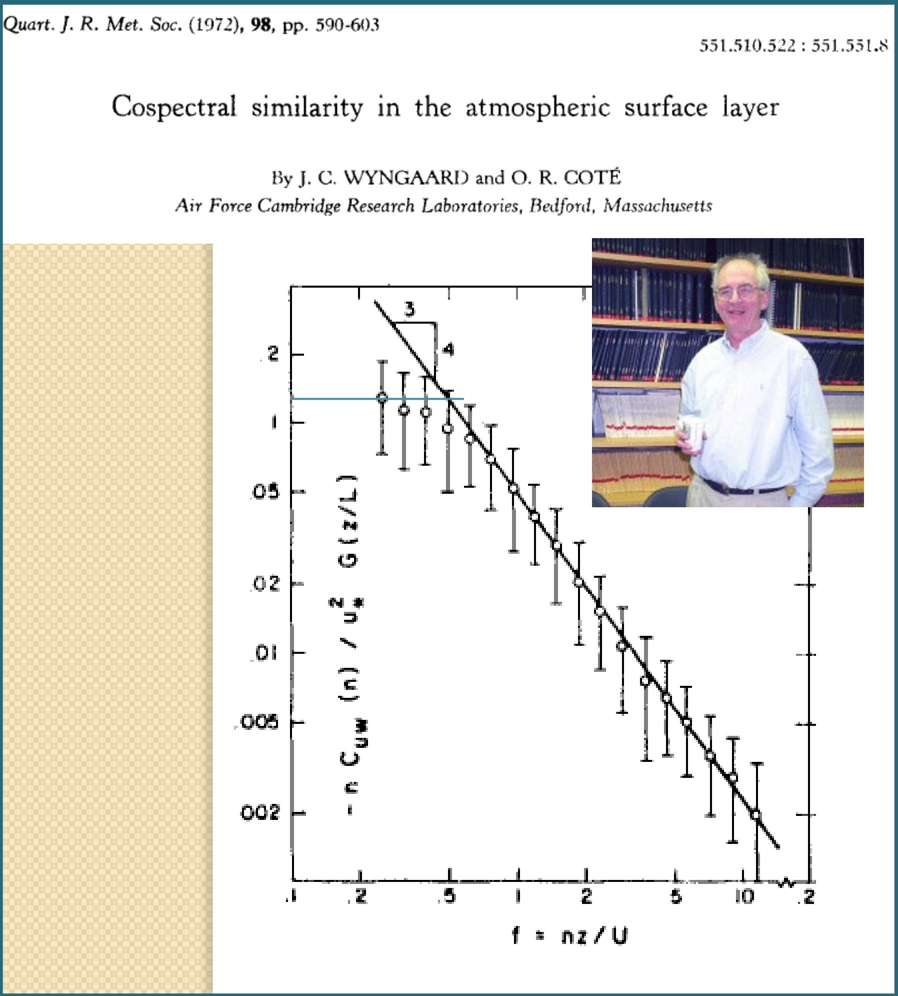
Co-spectral budget

$$C_{uw} = \overset{\text{Accepted range}}{0.15 - 0.16} = \left(\frac{1 - \overset{3/5}{\overset{\downarrow}{C_{IU}}}}{\underset{\overset{\uparrow}{1.8}}{A_u}}\overset{(24/55)(1.5)}{\overset{\swarrow}{C_o}}\right) = 0.145$$

This result establishes a link between the similarity constant in the *Lumley’s co-spectrum* and the ‘*collage*’ of well-established constants in turbulence.

Support for the -7/3 co-spectral exponent

Field Experiments (Kansas)



Quart. J. R. Met. Soc. (1972), **98**, pp. 590-603

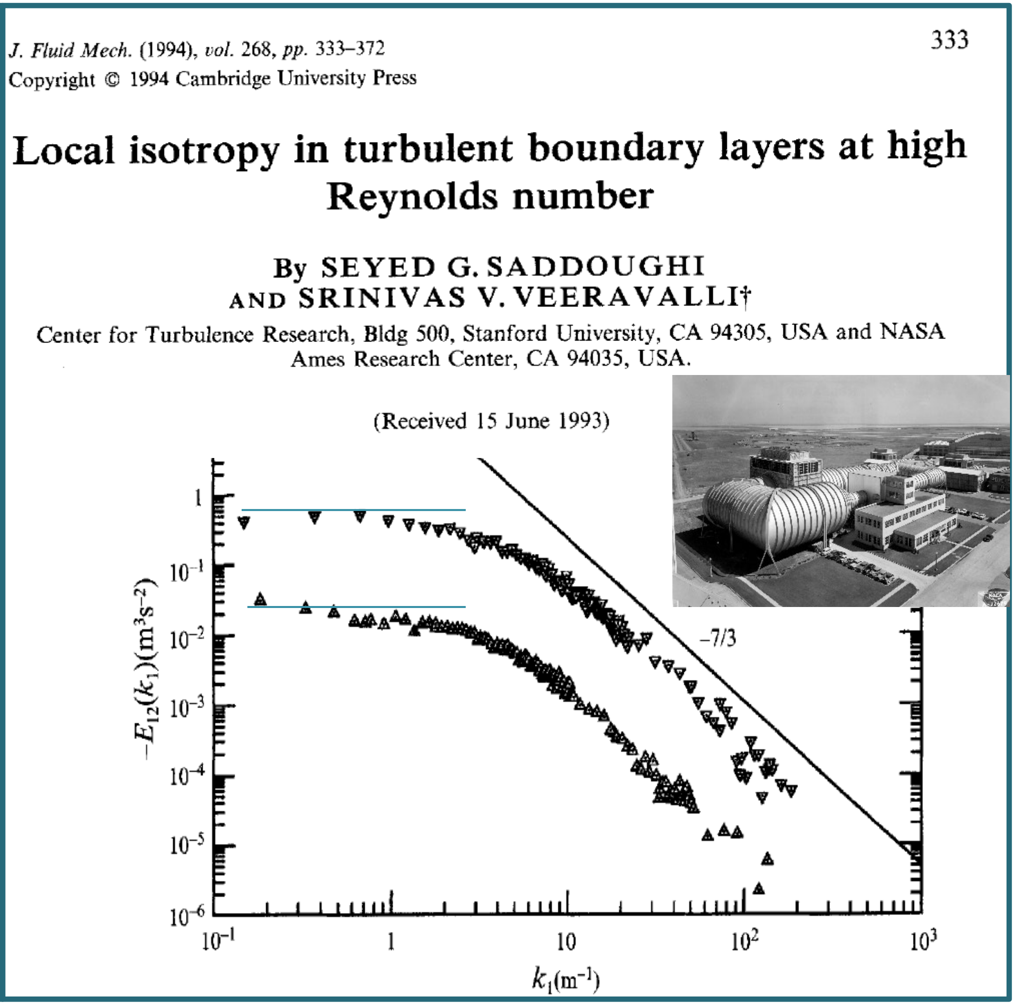
551.510.522 : 551.551.8

Cospectral similarity in the atmospheric surface layer

By J. C. WYNGAARD and O. R. COTÉ

Air Force Cambridge Research Laboratories, Bedford, Massachusetts

Lab Experiments (NASA)



J. Fluid Mech. (1994), *vol.* 268, *pp.* 333–372

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Local isotropy in turbulent boundary layers at high Reynolds number

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